the value of the function remains the same. The function $f(t)$ is then periodic and its period, T, is given by

$$\boxed{T = \frac{2\pi}{\omega}} \tag{14.3b}$$

Thus, the function $f(t)$ is periodic with period T,

$$f(t) = f(t+T)$$

The same result is obviously correct if we consider a sine function, $f(t) = A \sin \omega t$. Further, a linear combination of sine and cosine functions like,

$$f(t) = A \sin \omega t + B \cos \omega t \tag{14.3c}$$

is also a periodic function with the same period T. Taking,

$$A = D \cos \phi \text{ and } B = D \sin \phi$$

Eq. (14.3c) can be written as,

$$f(t) = D \sin (\omega t + \phi), \tag{14.3d}$$

Here D and ϕ are constant given by

$$D = \sqrt{A^2 + B^2} \text{ and } \phi = \tan^{-1}\left(\frac{B}{A}\right)$$

The great importance of periodic sine and cosine functions is due to a remarkable result proved by the French mathematician, Jean Baptiste Joseph Fourier (1768–1830): **Any periodic function can be expressed as a superposition of sine and cosine functions of different time periods with suitable coefficients**.

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Example 14.2 Which of the following functions of time represent (a) periodic and (b) non-periodic motion? Give the period for each case of periodic motion [ω is any positive constant].

(i) $\sin \omega t + \cos \omega t$
(ii) $\sin \omega t + \cos 2\,\omega t + \sin 4\,\omega t$
(iii) $e^{-\omega t}$
(iv) $\log(\omega t)$

Answer

(i) $\sin \omega t + \cos \omega t$ is a periodic function, it can also be written as $\sqrt{2}\,\sin(\omega t + \pi/4)$.

Now $\sqrt{2}\,\sin(\omega t + \pi/4) = \sqrt{2}\,\sin(\omega t + \pi/4 + 2\pi)$

$= \sqrt{2}\,\sin[\omega(t + 2\pi/\omega) + \pi/4]$

The periodic time of the function is $2\pi/\omega$.

(ii) This is an example of a periodic motion. It can be noted that each term represents a periodic function with a different angular frequency. Since period is the least interval of time after which a function repeats its value, $\sin \omega t$ has a period $T_0 = 2\pi/\omega$; $\cos 2\,\omega t$ has a period $\pi/\omega = T_0/2$; and $\sin 4\,\omega t$ has a period $2\pi/4\omega = T_0/4$. The period of the first term is a multiple of the periods of the last two terms. Therefore, the smallest interval of time after which the sum of the three terms repeats is T_0, and thus, the sum is a periodic function with a period $2\pi/\omega$.

(iii) The function $e^{-\omega t}$ is not periodic, it decreases monotonically with increasing time and tends to zero as $t \to \infty$ and thus, never repeats its value.

(iv) The function $\log(\omega t)$ increases monotonically with time t. It, therefore, never repeats its value and is a non-periodic function. It may be noted that as $t \to \infty$, $\log(\omega t)$ diverges to ∞. It, therefore, cannot represent any kind of physical displacement.

14.3 SIMPLE HARMONIC MOTION

Consider a particle oscillating back and forth about the origin of an x-axis between the limits $+A$ and $-A$ as shown in Fig. 14.3. This oscillatory motion is said to be simple harmonic if the displacement x of the particle from the origin varies with time as :

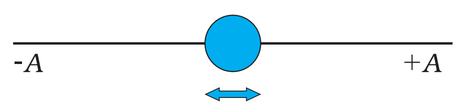

Fig. 14.3 *A particle vibrating back and forth about the origin of x-axis, between the limits +A and –A.*

$$x(t) = A \cos(\omega t + \phi) \tag{14.4}$$

where A, ω and ϕ are constants.

Thus, simple harmonic motion (SHM) is not any periodic motion but one in which displacement is a sinusoidal function of time.

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Fig. 14.4 shows the positions of a particle executing SHM at discrete value of time, each interval of time being $T/4$, where T is the period