Imp

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F_o \cos \omega_d t \qquad (14.37b)$$

This is the equation of an oscillator of mass m on which a periodic force of (angular) frequency ω_d is applied. The oscillator, initially, oscillates with its natural frequency ω. When we apply the external periodic force, the oscillations with the natural frequency die out, and then the body oscillates with the (angular) frequency of the external periodic force. Its displacement, after the natural oscillations die out, is given by

$$x(t) = A \cos (\omega_d t + \phi) \qquad (14.38)$$

where t is the time measured from the moment when we apply the periodic force.

The amplitude A is a function of the forced frequency ω_d and the natural frequency ω. Analysis shows that it is given by

Imp

$$\boxed{A = \frac{F_o}{\left\{m^2\left(\omega^2 - \omega_d^2\right)^2 + \omega_d^2 b^2\right\}^{1/2}}} \qquad (14.39a)$$

and $\boxed{\tan \phi = \frac{-v_o}{\omega_d x_o}}$ (14.39b)

where m is the mass of the particle and v_0 and x_0 are the velocity and the displacement of the particle at time $t = 0$, which is the moment when we apply the periodic force. Equation (14.39) shows that the amplitude of the forced oscillator depends on the (angular) frequency of the driving force. We can see a different behaviour of the oscillator when ω_d is far from ω and when it is close to ω. We consider these two cases.

(a) **Small Damping, Driving Frequency far from Natural Frequency :** In this case, $\omega_d b$ will be much smaller than $m(\omega^2 - \omega_d^2)$, and we can neglect that term. Then Eq. (14.39) reduces to

$\omega_d b$ can be neglected

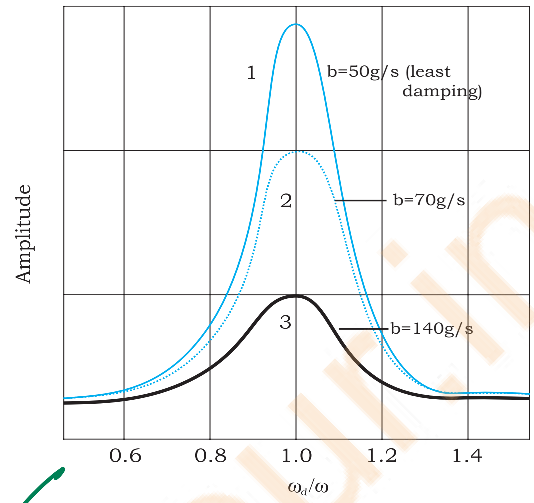
$$\boxed{A = \frac{F_o}{m\left(\omega^2 - \omega_d^2\right)}} \qquad (14.40)$$

For $\omega_d = \omega$, $A = A^{max}$
It is called 'Resonance'.

Fig. 14.21 shows the dependence of the displacement amplitude of an oscillator on the angular frequency of the driving force for different amounts of damping present in the system. It may be noted that in all cases the amplitude is the greatest when $\omega_d / \omega = 1$. The curves in this figure show that smaller the damping, the taller and narrower is the resonance peak.

In absence of damping $b=0$, for $\omega_d = \omega$
$A = A^{max} = \infty$

In presence of damping and for $\omega_d = \omega$, we get
$A = A^{max} = \frac{F_o}{\omega_d b}$
i.e $A^{max} \propto \frac{1}{b}$



In fig $b_1 < b_2 < b_3$
⇓
$A_1^{max} > A_2^{max} > A_3^{max}$
$\left(A^{max} \propto \frac{1}{b}\right)$

Fig. 14.21 *The displacement amplitude of a forced oscillator as a function of the angular frequency of the driving force. The amplitude is the greatest at $\omega_d / \omega = 1$, the resonance condition. The three curves correspond to different extents of damping present in the system. The curves 1 and 3 correspond to minimum and maximum damping in the system.*

$\frac{\omega_d}{\omega} = 1$ → $\omega_d = \omega$

If we go on changing the driving frequency, the amplitude tends to infinity when it equals the natural frequency. But this is the ideal case of zero damping, a case which never arises in a real system as the damping is never perfectly zero. You must have experienced in a swing that when the timing of your push exactly matches with the time period of the swing, your swing gets the maximum amplitude. This amplitude is large, but not infinity, because there is always some damping in your swing. This will become clear in the (b).

$A^{max} = \infty$ for $b = 0$

(b) **Driving Frequency Close to Natural Frequency :** If ω_d is very close to ω, $m(\omega^2 - \omega_d^2)$ would be much less than $\omega_d b$, for any reasonable value of b, then Eq. (14.39) reduces to

$$A = \frac{F_o}{\omega_d b} \qquad (14.41)$$

if $\omega_d \approx \omega \Rightarrow m(\omega^2 - \omega_d^2)$ can be neglected

This makes it clear that the maximum possible amplitude for a given driving frequency is governed by the driving frequency and the damping, and is never infinity. The phenomenon of increase in amplitude when the driving force is close to the natural frequency of the oscillator is called **resonance**.

Def. of Resonance

$\omega_d \approx \omega$

In our daily life, we encounter phenomena which involve resonance. Your experience with