b = 0, means there is no damping force present.

Of course, as expected, if we put b = 0, all equations of a damped oscillator in this section reduce to the corresponding equations of an undamped oscillator.

Example 14.10 For the damped oscillator shown in Fig. 14.19, the mass m of the block is 200 g, k = 90 N m^{-1} and the damping constant b is 40 g s^{-1}. Calculate (a) the period of oscillation, (b) time taken for its amplitude of vibrations to drop to half of its initial value, and (c) the time taken for its mechanical energy to drop to half its initial value.

Answer (a) We see that km = 90×0.2 = 18 kg N m^{-1} = kg^2 s^{-2}; therefore $\sqrt{km}$ = 4.243 kg s^{-1}, and b = 0.04 kg s^{-1}. Therefore, b is much less than $\sqrt{km}$. Hence, the time period T from Eq. (14.34) is given by

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$= 2\pi\sqrt{\frac{0.2 \text{ kg}}{90 \text{ N m}^{-1}}}$$

$$= 0.3 \text{ s}$$

(b) Now, from Eq. (14.33), the time, $T_{1/2}$, for the amplitude to drop to half of its initial value is given by,

$$T_{1/2} = \frac{\ln(1/2)}{b/2m}$$

$$= \frac{0.693}{40} \times 2 \times 200 \text{ s}$$

$$= 6.93 \text{ s}$$

(c) For calculating the time, $t_{1/2}$, for its mechanical energy to drop to half its initial value we make use of Eq. (14.35). From this equation we have,

$$E(t_{1/2})/E(0) = \exp(-bt_{1/2}/m)$$

Or $$\tfrac{1}{2} = \exp(-bt_{1/2}/m)$$

$$\ln(1/2) = -(bt_{1/2}/m)$$

Or $$t_{1/2} = \frac{0.693}{40 \text{ g s}^{-1}} \times 200 \text{ g}$$

$$= 3.46 \text{ s}$$

This is just half of the decay period for amplitude. This is not surprising, because, according to Eqs. (14.33) and (14.35), energy depends on the square of the amplitude. Notice that there is a factor of 2 in the exponents of the two exponentials.

14.10 FORCED OSCILLATIONS AND RESONANCE

When a system (such as a simple pendulum or a block attached to a spring) is displaced from its equilibrium position and released, it oscillates with its natural frequency ω, and the oscillations are called **free oscillations.** All free oscillations eventually die out because of the ever present damping forces. However, an external agency can maintain these oscillations. These are called **forced** or **driven** oscillations. We consider the case when the external force is itself periodic, with a frequency ω_d called the driven frequency. The most important fact of forced periodic oscillations is that the system oscillates not with its natural frequency ω, but at the frequency ω_d of the external agency; the free oscillations die out due to damping. The most familiar example of forced oscillation is when a child in a garden swing periodically presses his feet against the ground (or someone else periodically gives the child a push) to maintain the oscillations.

Suppose an external force $F(t)$ of amplitude F_0 that varies periodically with time is applied to a damped oscillator. Such a force can be represented as,

Periodic driving force →

$$F(t) = F_o \cos \omega_d t \tag{14.36}$$

The motion of a particle under the combined action of a linear restoring force, damping force and a time dependent driving force represented by Eq. (14.36) is given by,

Net force →

$$\boxed{m\,a(t) = -k\,x(t) - bv(t) + F_o \cos \omega_d t} \tag{14.37a}$$

Substituting d^2x/dt^2 for acceleration in Eq. (14.37a) and rearranging it, we get

1 → linear restoring force

2 → damping force

3 → periodic driving force