The damping force depends on the nature of the surrounding medium. If the block is immersed in a liquid, the magnitude of damping will be much greater and the dissipation of energy much faster. The damping force is generally proportional to velocity of the bob. [Remember Stokes' Law, Eq. (10.19)] and acts opposite to the direction of velocity. If the damping force is denoted by $\mathbf{F}_d$, we have

'b' is called the damping constant

$$\mathbf{F}_d = -b\,\mathbf{v} \tag{14.30}$$

where the positive constant b depends on characteristics of the medium (viscosity, for example) and the size and shape of the block, etc. Eq. (14.30) is usually valid only for small velocity.

When the mass m is attached to the spring (hung vertically as shown in Fig. 14.19) and released, the spring will elongate a little and the mass will settle at some height. This position, shown by O in Fig 14.19, is the equilibrium position of the mass. If the mass is pulled down or pushed up a little, the restoring force on the block due to the spring is $\mathbf{F}_S = -k\mathbf{x}$, where $\mathbf{x}$ is the displacement* of the mass from its equilibrium position. Thus, the total force acting on the mass at any time t, is $\mathbf{F} = -k\mathbf{x} - b\mathbf{v}$.

If $\mathbf{a}(t)$ is the acceleration of mass at time t, then by Newton's Law of Motion applied along the direction of motion, we have

$$m\,a(t) = -k\,x(t) - b\,v(t) \tag{14.31}$$

$a = \frac{d^2x}{dt^2}$

$v = \frac{dx}{dt}$

Here we have dropped the vector notation because we are discussing one-dimensional motion.

Using the first and second derivatives of x (t) for v (t) and a (t), respectively, we have

$$\boxed{m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + k\,x = 0} \tag{14.32}$$

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The solution of Eq. (14.32) describes the motion of the block under the influence of a damping force which is proportional to velocity. The solution is found to be of the form

$$\boxed{x(t) = A\,e^{-b\,t/2m}\cos(\omega' t + \phi)} \tag{14.33}$$

where A is the amplitude and ω' is the angular frequency of the damped oscillator given by,

$$\boxed{\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}} \tag{14.34}$$

In this function, the cosine function has a period $2\pi/\omega'$ but the function $x(t)$ is not strictly periodic because of the factor $e^{-b\,t/2m}$ which decreases continuously with time. However, if the decrease is small in one time period T, the motion represented by Eq. (14.33) is approximately periodic.

The solution, Eq. (14.33), can be graphically represented as shown in Fig. 14.20. We can regard it as a cosine function whose amplitude, which is $Ae^{-b\,t/2m}$, gradually decreases with time.

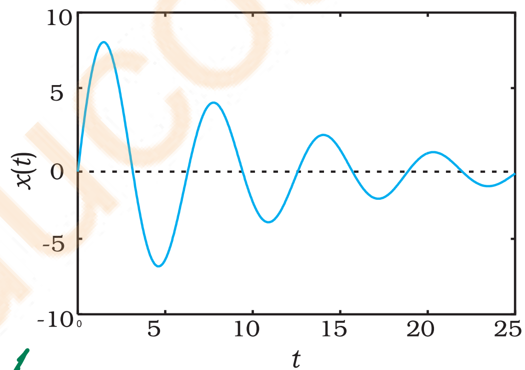

Fig. 14.20 A damped oscillator is approximately periodic with decreasing amplitude of oscillation. With greater damping, oscillations die out faster.

Now the mechanical energy of the undamped oscillator is $1/2\ kA^2$. For a damped oscillator, the amplitude is not constant but depends on time. For small damping, we may use the same expression but regard the amplitude as $A\,e^{-bt/2m}$.

$$\boxed{E(t) = \frac{1}{2}k\,A^2\,e^{-b\,t/m}} \tag{14.35}$$

Equation (14.35) shows that the total energy of the system decreases exponentially with time. Note that small damping means that the dimensionless ratio $\left(\frac{b}{\sqrt{km}}\right)$ is much less than 1.

* *Under gravity, the block will be at a certain equilibrium position O on the spring; x here represents the displacement from that position.*