of the function sin θ . From this table it can be seen that for θ as large as 20 degrees, sin θ is nearly the same as θ **expressed in radians**.

Table 14.1 **sin θ as a function of angle θ**

θ (degrees)	θ (radians)	sin θ
0	0	0
5	0.087	0.087
10	0.174	0.174
15	0.262	0.259
20	0.349	0.342

Equation (14.27) is mathematically, identical to Eq. (14.11) except that the variable is angular displacement. Hence we have proved that for small θ, the motion of the bob is simple harmonic. From Eqs. (14.27) and (14.11),

$$\boxed{\omega = \sqrt{\frac{mgL}{I}}}$$

and

$$\boxed{T = 2\pi\sqrt{\frac{I}{mgL}}} \qquad (14.28)$$

Now since the string of the simple pendulum is massless, the moment of inertia I is simply mL^2. Eq. (14.28) then gives the well-known formula for time period of a simple pendulum.

$$\boxed{T = 2\pi\sqrt{\frac{L}{g}}} \qquad (14.29)$$

Example 14.9 What is the length of a simple pendulum, which ticks seconds ?

Answer From Eq. (14.29), the time period of a simple pendulum is given by,

$$T = 2\pi\sqrt{\frac{L}{g}}$$

From this relation one gets,

$$L = \frac{gT^2}{4\pi^2}$$

The time period of a simple pendulum, which ticks seconds, is 2 s. Therefore, for $g = 9.8$ m s^{-2} and $T = 2$ s, L is

$$= \frac{9.8(\text{m s}^{-2}) \times 4(\text{s}^2)}{4\pi^2}$$

$$= 1 \text{ m}$$

14.9 DAMPED SIMPLE HARMONIC MOTION

We know that the motion of a simple pendulum, swinging in air, dies out eventually. Why does it happen ? This is because the air drag and the friction at the support oppose the motion of the pendulum and dissipate its energy gradually. Reason The pendulum is said to execute **damped oscillations**. In dampled oscillations, the energy of the system is dissipated continuously; but, for small damping, the oscillations remain approximately periodic. The dissipating forces are generally the frictional forces. To understand the effect of such external forces on the motion of an oscillator, let us consider a system as shown in Fig. 14.19. Here a block of mass m connected to an elastic spring of spring constant k oscillates vertically. If the block is pushed down a little and released, its angular frequency of oscillation is $\omega = \sqrt{\frac{k}{m}}$, as seen in Eq. (14.20). However, in practice, the surrounding medium (air) will exert a damping force on the motion of the block and the mechanical energy of the block-spring system will decrease. The energy loss will appear as heat of the surrounding medium (and the block also) [Fig. 14.19].

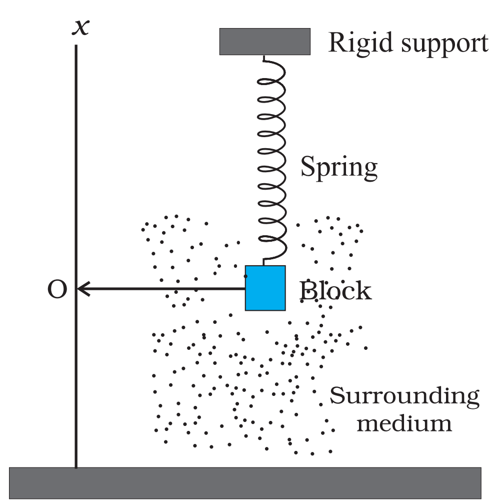

Fig. 14.19 *The viscous surrounding medium exerts a damping force on an oscillating spring, eventually bringing it to rest.*