Let θ be the angle made by the string with the vertical. When the bob is at the mean position, $\theta = 0$

There are only two forces acting on the bob; the tension T along the string and the vertical force due to gravity (=mg). The force *mg* can be resolved into the component *mg* cosθ along the string and *mg* sinθ perpendicular to it. Since the motion of the bob is along a circle of length *L* and centre at the support point, the bob has a radial acceleration ($\omega^2 L$) and also a tangental acceleration; the latter arises since motion along the arc of the circle is not uniform. The radial acceleration is provided by the net radial force T –*mg* cosθ, while the tangential acceleration is provided by *mg* sinθ. It is more convenient to work with torque about the support since the radial force gives zero torque. Torque τ about the support is entirely provided by the tangental component of force

$$\tau = -L\,(mg\sin\theta) \tag{14.22}$$

This is the restoring torque that tends to reduce angular displacement — hence the negative sign. By Newton's law of rotational motion,

$$\tau = I\,\alpha \tag{14.23}$$

where *I* is the moment of inertia of the system about the support and α is the angular acceleration. Thus,

$$I\,\alpha = -m\,g\sin\theta\;\;L \tag{14.24}$$

Or,

$$\alpha = -\frac{m\,g\,L}{I}\sin\theta \tag{14.25}$$

We can simplify Eq. (14.25) if we assume that the displacement θ is small. We know that sin θ can be expressed as,

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} \pm \ldots \tag{14.26}$$

where θ is in radians.

Now if θ is small, sin θ can be approximated by θ and Eq. (14.25) can then be written as,

$$\alpha = -\frac{mgL}{I}\theta \tag{14.27}$$

In Table 14.1, we have listed the angle θ in degrees, its equivalent in radians, and the value

SHM - how small should the amplitude be?

When you perform the experiment to determine the time period of a simple pendulum, your teacher tells you to keep the amplitude small. But have you ever asked how small is small? Should the amplitude to 5^0, 2^0, 1^0, or 0.5^0? Or could it be 10^0, 20^0, or 30^0?

To appreciate this, it would be better to measure the time period for different amplitudes, up to large amplitudes. Of course, for large oscillations, you will have to take care that the pendulum oscillates in a vertical plane. Let us denote the time period for small-amplitude oscillations as $T(0)$ and write the time period for amplitude θ_0 as $T(\theta_0) = cT(0)$, where c is the multiplying factor. If you plot a graph of c versus θ_0, you will get values somewhat like this:

θ_0	:	20^0	45^0	50^0	70^0	90^0
c	:	1.02	1.04	1.05	1.10	1.18

This means that the error in the time period is about 2% at an amplitude of 20^0, 5% at an amplitude of 50^0, and 10% at an amplitude of 70^0 and 18% at an amplitude of 90^0.

In the experiment, you will never be able to measure $T(0)$ because this means there are no oscillations. Even theoretically, sin θ is exactly equal to θ only for $\theta = 0$. There will be some inaccuracy for all other values of θ. The difference increases with increasing θ. Therefore we have to decide how much error we can tolerate. No measurement is ever perfectly accurate. You must also consider questions like these: What is the accuracy of the stopwatch? What is your own accuracy in starting and stopping the stopwatch? You will realise that the accuracy in your measurements at this level is never better than 5% or 10%. Since the above table shows that the time period of the pendulum increases hardly by 5% at an amplitude of 50^0 over its low amplitude value, you could very well keep the amplitude to be 50° in your experiments.