Example 14.8 A 5 kg collar is attached to a spring of spring constant 500 N m^{-1}. It slides without friction over a horizontal rod. The collar is displaced from its equilibrium position by 10.0 cm and released. Calculate
(a) the period of oscillation,
(b) the maximum speed and
(c) maximum acceleration of the collar.

Answer (a) The period of oscillation as given by Eq. (14.21) is,

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{5.0\ \text{kg}}{500\ \text{N m}^{-1}}}$$
$$= (2\pi/10)\ \text{s}$$
$$= 0.63\ \text{s}$$

(b) The velocity of the collar executing SHM is given by,

$$v(t) = -A\omega \sin(\omega t + \phi)$$

The maximum speed is given by,

$$v_m = A\omega$$
$$= 0.1 \times \sqrt{\frac{k}{m}}$$
$$= 0.1 \times \sqrt{\frac{500\ \text{N m}^{-1}}{5\ \text{kg}}}$$
$$= 1\ \text{m s}^{-1}$$

and it occurs at $x = 0$

(c) The acceleration of the collar at the displacement $x(t)$ from the equilibrium is given by,

$$a(t) = -\omega^2 x(t)$$
$$= -\frac{k}{m}\, x(t)$$

Therefore, the maximum acceleration is,

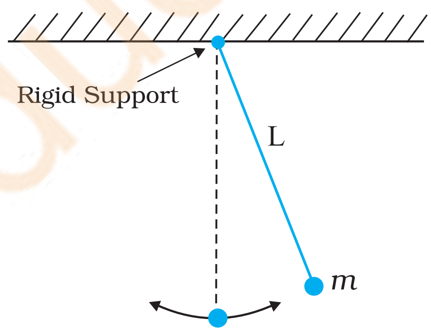
$$a_{max} = \omega^2 A$$
$$= \frac{500\ \text{N m}^{-1}}{5\ \text{kg}} \times 0.1\ \text{m}$$
$$= 10\ \text{m s}^{-2}$$

and it occurs at the extremities. ◀

14.8.2 The Simple Pendulum

It is said that Galileo measured the periods of a swinging chandelier in a church by his pulse beats. He observed that the motion of the chandelier was periodic. The system is a kind of pendulum. You can also make your own pendulum by tying a piece of stone to a long unstretchable thread, approximately 100 cm long. Suspend your pendulum from a suitable support so that it is free to oscillate. Displace the stone to one side by a small distance and let it go. The stone executes a to and fro motion, it is periodic with a period of about two seconds.

We shall show that this periodic motion is simple harmonic for small displacements from the mean position. Consider simple pendulum — a small bob of mass m tied to an inextensible massless string of length L. The other end of the string is fixed to a rigid support. The bob oscillates in a plane about the vertical line through the support. Fig. 14.18(a) shows this system. Fig. 14.18(b) is a kind of 'free-body' diagram of the simple pendulum showing the forces acting on the bob.



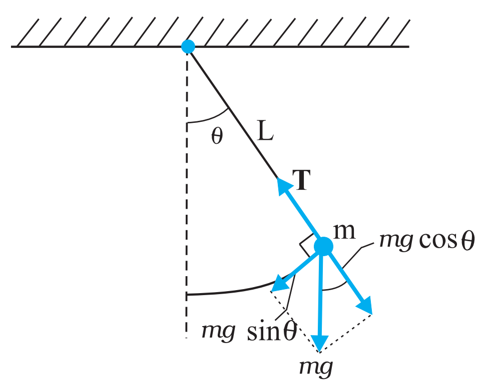


Fig. 14.18 *(a) A bob oscillating about its mean position. (b) The radial force T-mg cosθ provides centripetal force but no torque about the support. The tangential force mg sinθ provides the restoring torque.*