It follows from Eqs. (14.15) and (14.17) that the total energy, E, of the system is,

$$E = U + K$$

$$= \frac{1}{2} k\,A^2 \cos^2(\omega t + \phi) + \frac{1}{2} k\,A^2 \sin^2(\omega t + \phi)$$

$$= \frac{1}{2} k\,A^2 \left[\cos^2(\omega t + \phi) + \sin^2(\omega t + \phi)\right]$$

Using the familiar trigonometric identity, the value of the expression in the brackets is unity. Thus,

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where $k = m\omega^2$

$$\boxed{E = \frac{1}{2} k\,A^2} \qquad (14.18)$$

The total mechanical energy of a harmonic oscillator is thus independent of time as expected for motion under any conservative force. The time and displacement dependence of the potential and kinetic energies of a linear simple harmonic oscillator are shown in Fig. 14.16.

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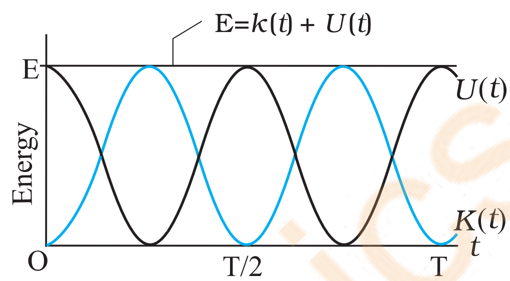

(a)

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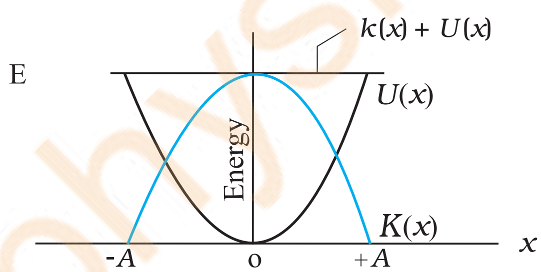

(b)

Fig. 14.16 *Kinetic energy, potential energy and total energy as a function of time [shown in (a)] and displacement [shown in (b)] of a particle in SHM. The kinetic energy and potential energy both repeat after a period T/2. The total energy remains constant at all t or x.*

Observe that both kinetic energy and potential energy in SHM are seen to be always positive in Fig. 14.16. Kinetic energy can, of course, be never negative, since it is proportional to the square of speed. Potential energy is positive by choice of the undermined constant in potential energy. Both kinetic energy and potential energy peak twice during each period of SHM. For $x = 0$, the energy is kinetic; at the extremes $x = \pm A$, it is all potential energy. In the course of motion between these limits, kinetic energy increases at the expense of potential energy or vice-versa.

Example 14.7 A block whose mass is 1 kg is fastened to a spring. The spring has a spring constant of 50 N m^{-1}. The block is pulled to a distance x = 10 cm from its equilibrium position at $x = 0$ on a frictionless surface from rest at t = 0. Calculate the kinetic, potential and total energies of the block when it is 5 cm away from the mean position.

Answer The block executes SHM, its angular frequency, as given by Eq. (14.14b), is

$$\omega = \sqrt{\frac{k}{m}}$$

$$= \sqrt{\frac{50\ \mathrm{N\ m^{-1}}}{1\mathrm{kg}}}$$

$$= 7.07\ \mathrm{rad\ s^{-1}}$$

Its displacement at any time t is then given by,

$$x(t) = 0.1 \cos(7.07t)$$

Therefore, when the particle is 5 cm away from the mean position, we have

$$0.05 = 0.1 \cos(7.07t)$$

Or $\quad \cos(7.07t) = 0.5$ and hence

$$\sin(7.07t) = \frac{\sqrt{3}}{2} = 0.866$$