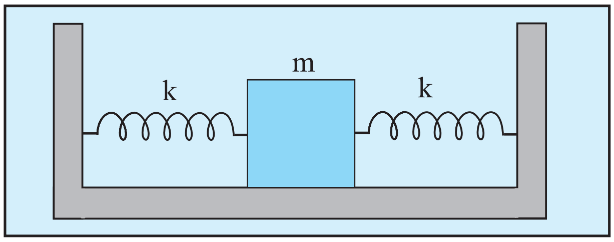

Fig. 14.14

Answer Let the mass be displaced by a small distance x to the right side of the equilibrium position, as shown in Fig. 14.15. Under this situation the spring on the left side gets

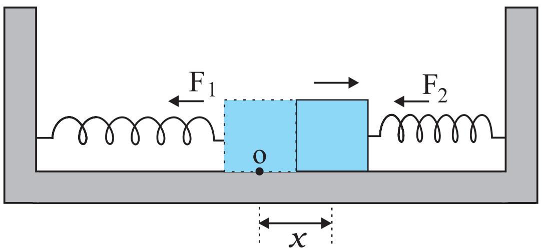

Fig. 14.15

elongated by a length equal to x and that on the right side gets compressed by the same length. The forces acting on the mass are then,

$F_1 = -k\,x$ (force exerted by the spring on the left side, trying to pull the mass towards the mean position)

$F_2 = -k\,x$ (force exerted by the spring on the right side, trying to push the mass towards the mean position)

The net force, F, acting on the mass is then given by,

$$F = -2kx$$

Hence the force acting on the mass is proportional to the displacement and is directed towards the mean position; therefore, the motion executed by the mass is simple harmonic. The time period of oscillations is,

$$T = 2\pi \sqrt{\frac{m}{2k}}$$

14.7 ENERGY IN SIMPLE HARMONIC MOTION

Both kinetic and potential energies of a particle in SHM vary between zero and their maximum values.

In section 14.5 we have seen that the velocity of a particle executing SHM, is a periodic function of time. It is zero at the extreme positions of displacement. Therefore, the kinetic energy (K) of such a particle, which is defined as

$$K = \frac{1}{2} m v^2$$

$$= \frac{1}{2} m\,\omega^2 A^2 \sin^2(\omega t + \phi)$$

where $k = m\omega^2$

$$\boxed{K = \frac{1}{2} k\,A^2 \sin^2(\omega t + \phi)} \qquad (14.15)$$

is also a periodic function of time, being zero when the displacement is maximum and maximum when the particle is at the mean position. Note, since the sign of v is immaterial in K, the period of K is $T/2$.

What is the potential energy (U) of a particle executing simple harmonic motion? In Chapter 6, we have seen that the concept of potential energy is possible only for conservative forces. The spring force $F = -kx$ is a conservative force, with associated potential energy

$$U = \frac{1}{2} k\,x^2 \qquad (14.16)$$

Hence the potential energy of a particle executing simple harmonic motion is,

$$U(x) = \frac{1}{2} k\,x^2$$

where $k = m\omega^2$

$$\boxed{U(t) = \frac{1}{2} k\,A^2 \cos^2(\omega t + \phi)} \qquad (14.17)$$

Thus, the potential energy of a particle executing simple harmonic motion is also periodic, with period $T/2$, being zero at the mean position and maximum at the extreme displacements.