

Theorem:

When a moving body makes oblique elastic collision with another body of equal mass at rest, after the collision they move in mutually perpendicular direction.

Proof:

If the bodies are of equal mass, i.e. if $m_1 = m_2 = m$ (let)

$$\text{By eq. (xi)} \quad u_1 = v_1 \cos \theta_1 + v_2 \cos \theta_2 \quad \text{----- (xiv)}$$

$$\text{By (xii)} \quad 0 = v_1 \sin \theta_1 + v_2 \sin \theta_2 \quad \text{----- (xv)}$$

$$\text{And by eq. (xiii)} \quad u_1^2 = v_1^2 + v_2^2 \quad \text{----- (xvi)}$$

$$\text{By (xiv)}^2 + \text{(xv)}^2$$

$$u_1^2 = [v_1^2 \cos^2 \theta_1 + v_2^2 \cos^2 \theta_2 + 2v_1 \cos \theta_1 v_2 \cos \theta_2] + \\ [v_1^2 \sin^2 \theta_1 + v_2^2 \sin^2 \theta_2 - 2v_1 v_2 \sin \theta_1 \sin \theta_2]$$

$$u_1^2 = v_1^2 [\cos^2 \theta_1 + \sin^2 \theta_1] + v_2^2 [\cos^2 \theta_2 + \sin^2 \theta_2] \\ + 2v_1 v_2 \{ \cos \theta_1 \cos \theta_2 - \sin \theta_1 \sin \theta_2 \}$$

$$u_1^2 = v_1^2 + v_2^2 + 2v_1 v_2 \cos (\theta_1 + \theta_2)$$

$$u_1^2 = u_1^2 + 2v_1 v_2 \cos (\theta_1 + \theta_2)$$

$$2v_1 v_2 \cos [\theta_1 + \theta_2] = 0$$

$$\therefore 2v_1 v_2 \neq 0, \text{ we get } \cos [\theta_1 + \theta_2] = 0$$

$$\text{So,} \quad \theta_1 + \theta_2 = 90^\circ$$

i.e. When a moving body makes oblique elastic collision with another body of equal mass at rest, after the collision they move in mutually perpendicular direction.