If v_B^{min} be the critical velocity of the particle at the bottom, then from conservation of energy

$$K_A + U_A = K_B + U_B$$

$$mg(2\ell) + \frac{1}{2}m(v_A^{min})^2 = \frac{1}{2}m(v_B^{min})^2 + 0$$

$$\Rightarrow \quad 2mg\ell + \frac{1}{2}mgl = \frac{1}{2}m(v_B^{min})^2 \qquad (\because v_b^{min} = \sqrt{g\ell})$$

$$\Rightarrow \quad v_B^{min} = \sqrt{5g\ell}$$

$$v_B^{min} = \sqrt{5gl} \quad \text{..........(6)}$$

If v_C^{min} be the critical velocity of the particle at the bottom, then from conservation of energy

$$K_A + U_A = K_C + U_C$$

$$mg(2\ell) + \frac{1}{2}m(v_A^{min})^2 = \frac{1}{2}m(v_C^{min})^2 + mg\ell$$

$$\Rightarrow \quad 2mg\ell + \frac{1}{2}mgl = \frac{1}{2}m(v_C^{min})^2 + mg\ell \qquad (\because v_b^{min} = \sqrt{g\ell})$$

$$\Rightarrow \quad v_C^{min} = \sqrt{3g\ell}$$

$$v_C^{min} = \sqrt{3gl} \quad \text{..........(7)}$$

Important Cases:

(a) If $v_B > \sqrt{5gl}$

In this case tension in the string will not be zero at any of the point, which implies that the particle will continue complete the circular motion.

(b) If $v_B = \sqrt{5gl}$:

In this case the tension at the top most point (B) will be zero, which implies that the particle will just complete the circular motion.