(IV) $S_{nth} = u + (1/2)\, a(2n-1)$

Let the body starts motion with initial velocity u at t = 0. Let the position of the body at t = (n-1) and t = n are x = x_{n-1} and x = x_n respectively. By definition $v = \frac{dx}{dt}$ Or $dx = v.dt$

Integrating both sides

$$\int_{x_{n-1}}^{x_n} dx = \int_{(n-1)}^{n} v.dt \quad Or \quad \int_{x_{n-1}}^{x_n} dx = \int_{(n-1)}^{n} (u + a.t).dt$$

$$\left[x\right]_{x_{n-1}}^{x_n} = \int_{(n-1)}^{n} u.dt + \int_{(n-1)}^{n} at.dt$$

$$\left[x\right]_{x_{n-1}}^{x_n} = u \int_{(n-1)}^{n} dt + a \int_{(n-1)}^{n} t.dt$$

OR $$x_n - x_{n-1} = u\left[t\right]_{(n-1)}^{n} + a\left[\frac{t^2}{2}\right]_{(n-1)}^{n}$$

$$S_{nth} = u\{n-(n-1)\} + a\left\{\frac{n^2}{2} - \frac{(n-1)^2}{2}\right\}$$

$$S_{nth} = u + \frac{1}{2}a\,(2n-1)$$