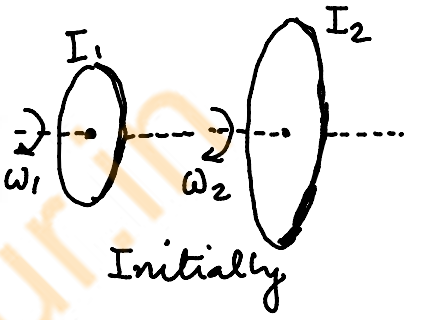
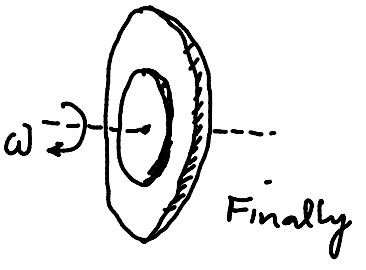
Two discs of moments of inertia I_1 and I_2 about their respective axes (normal to the disc and passing through the centre), and rotating with angular speeds ω_1 and ω_2 are brought into contact face to face with their axes of rotation coincident. (a) What is the angular speed of the two-disc system ? (b) Show that the kinetic energy of the combined system is less than the sum of the initial kinetic energies of the two discs. How do you account for this loss in energy ? Take $\omega_1 \neq \omega_2$.

rotate





$K_f < K_i$

(a) $\because \vec{\tau}^{ext} = 0 \Rightarrow$ COAM $\quad L_f = L_i$

$$I_f \omega_f = I_i \omega_i$$

$$(I_1 + I_2)\,\omega = I_1\omega_1 + I_2\omega_2$$

$$\omega = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2} \quad \text{— (1)}$$

(b) $$K_i = \frac{1}{2} I_1\omega_1^2 + \frac{1}{2} I_2\omega_2^2 \quad \text{— (2)}$$

$$I_f = \frac{1}{2}(I_1 + I_2)\omega^2 = \frac{1}{2}\cancel{(I_1 + I_2)} \cdot \left[\frac{(I_1\omega_1 + I_2\omega_2)^2}{\cancel{(I_1 + I_2)^2}}\right]$$