Relation Between Angular Momentum and Torque:

(a) **For a Single Particle:** $\vec{L} = \vec{r} \times \vec{p}$ — (1) $(\vec{p} = m\vec{v})$

$\vec{\tau} = \vec{r} \times \vec{F}$ — (2) $\left(\because \vec{F} = \frac{d\vec{p}}{dt}\right)$

Rate of change of angular momentum

$$\frac{d\vec{L}}{dt} = \frac{d}{dt}(\vec{r} \times \vec{p}) \qquad \left\{\frac{d(f.g)}{dx} = f\frac{dg}{dx} + \frac{df}{dx}.g\right\}$$

$$\frac{d\vec{L}}{dt} = \vec{r} \times \frac{d\vec{p}}{dt} + \frac{d\vec{r}}{dt} \times \vec{p} = \vec{r} \times \vec{F} + \vec{v} \times (m\vec{v})$$

$$\frac{d\vec{L}}{dt} = \vec{r} \times \vec{F} + m(\vec{v} \times \vec{v})$$

$$\frac{d\vec{L}}{dt} = \vec{r} \times \vec{F} \qquad (\because \vec{v} \times \vec{v} = 0)$$

$$\boxed{\frac{d\vec{L}}{dt} = \vec{\tau}} \quad \text{— (1)}$$

Relation Between angular Momentum and Torque for a Rigid Body: