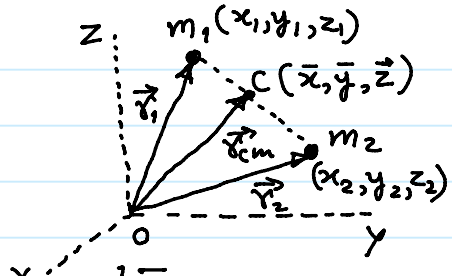
Class XI Centre of Mass

Friday, October 8, 2021 9:22 PM

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1+m_2} \quad \text{— (1)}$$



$$\vec{r}_1 = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$$

$$\vec{r}_2 = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$$

$$\vec{r}_{cm} = x_{cm}\hat{i} + y_{cm}\hat{j} + z_{cm}\hat{k}$$

by eq (1)

$$x_{cm}\hat{i} + y_{cm}\hat{j} + z_{cm}\hat{k} = \frac{1}{(m_1+m_2)}\{m_1(x_1\hat{i}+y_1\hat{j}+z_1\hat{k}) + m_2(x_2\hat{i}+y_2\hat{j}+z_2\hat{k})\}$$

$\bar{x} = x_{cm}$, $\bar{y} = y_{cm}$, $\bar{z} = z_{cm}$

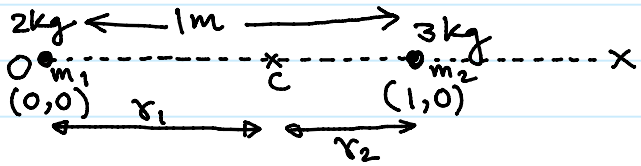
$$x_{cm}\hat{i} + y_{cm}\hat{j} + z_{cm}\hat{k} = \frac{1}{M}\{(m_1x_1 + m_2x_2)\hat{i} + (m_1y_1 + m_2y_2)\hat{j} + (m_1z_1 + m_2z_2)\hat{k}\}$$

$$x_{cm} = \frac{1}{M}(m_1x_1 + m_2x_2) \quad \text{— (2)} \quad \rightarrow \quad x_{cm} = \frac{1}{M}\sum_{i=1}^{n} m_ix_i$$

$$y_{cm} = \frac{1}{M}(m_1y_1 + m_2y_2) \quad \text{— (3)} \quad \rightarrow \quad y_{cm} = \frac{1}{M}\sum_{i=1}^{n} m_iy_i$$

$$z_{cm} = \frac{1}{M}(m_1z_1 + m_2z_2) \quad \text{— (4)} \quad \rightarrow \quad z_{cm} = \frac{1}{M}\sum m_iz_i$$

Ex1:



$m_1 = 2kg$, $m_2 = 3kg$

$x_1 = 0$, $x_2 = 1m$

$y_1 = 0$, $y_2 = 0$

$M = m_1 + m_2 = 5kg$

$$x_{cm} = \frac{1}{M}(m_1x_1 + m_2x_2)$$

$$= \frac{1}{5}(2\times0 + 3\times1)$$

$$x_{cm} = \frac{3}{5}m = 0.6m$$

$$y_{cm} = \frac{1}{5}(2\times0 + 3\times0) = \frac{0}{5}$$

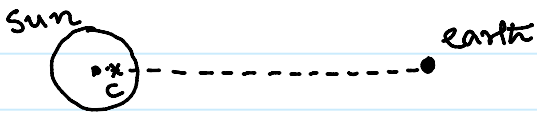
$y_{cm} = 0$

CM $\equiv (0.6m, 0)$

$r_1 = 0.6m$, $r_2 = 1 - 0.6m = 0.4m$

$$\frac{r_1}{r_2} = \frac{0.6}{0.4} = \frac{3}{2} \Rightarrow \boxed{\frac{r_1}{r_2} = \frac{m_2}{m_1}} \quad \text{— (5)} \rightarrow r \propto \frac{1}{m}$$

$$m_1r_1 = m_2r_2 \quad \text{— (6)}$$



Note: if $m_1 = m_2 \rightarrow$ by eq (5) $\frac{r_1}{r_2} = \frac{m_1}{m_1} = 1 \Rightarrow \boxed{r_1 = r_2}$

i.e CM of a system of two objects of equal mass lie at the centre of the line joining the two objects.

Ex2:

$m_1 = 2kg$, $\vec{r}_1 = \hat{i} - 2\hat{j} + 3\hat{k}$ m

$m_2 = 3kg$, $\vec{r}_2 = 2\hat{i} - 5\hat{j} + \hat{k}$ m

Find $\vec{r}_{cm}$ = ? $\rightarrow (1, -2, 3)$ m