

Derivation of $\vec{r}_{cm}$:

For the motion of 1st body, using Newton's 2nd

$$\checkmark \frac{d\vec{p}_1}{dt} = \vec{F}_{1net}$$

$$\frac{d}{dt}(m_1 \vec{v}_1) = \vec{F}_1 + \vec{F}_{12} \quad \text{--- (2)}$$

$$\frac{d}{dt}\left(m_1 \frac{d\vec{r}_1}{dt}\right) = \vec{F}_1 + \vec{F}_{12}$$

$$m_1 \frac{d^2 \vec{r}_1}{dt^2} = \vec{F}_1 + \vec{F}_{12} \quad \text{--- (3)}$$

acceleration of 1st body

For 2nd body

$$\frac{d\vec{p}_2}{dt} = \vec{F}_{2net}$$

$$m_2 \frac{d^2 \vec{r}_2}{dt^2} = \vec{F}_2 + \vec{F}_{21} \quad \text{--- (4)}$$

by (3) + (4)

$$m_1 \frac{d^2 \vec{r}_1}{dt^2} + m_2 \frac{d^2 \vec{r}_2}{dt^2} = \vec{F}_1 + \vec{F}_2 + \underbrace{\vec{F}_{12} + \vec{F}_{21}}_0$$

$$\frac{d^2}{dt^2} (m_1 \vec{r}_1 + m_2 \vec{r}_2) = \vec{F}_1 + \vec{F}_2 + 0$$

$$\begin{cases} \because \vec{F}_{12} = -\vec{F}_{21} \\ \vec{F}_{12} + \vec{F}_{21} = 0 \end{cases}$$

$$(m_1 + m_2) \frac{d^2}{dt^2} \left(\frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2} \right) = \vec{F}_{ext}$$

$$\text{Let } \vec{r}_{cm} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2}$$

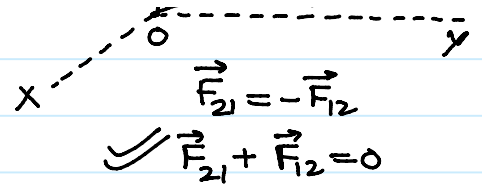
$$(m_1 + m_2) \frac{d^2 \vec{r}_{cm}}{dt^2} = \vec{F}_{ext} \quad \text{--- (5)}$$

Total mass of the system

Position vector of the point at which whole of the mass of the system is present

Total ext. force on the system

$\vec{r}_{cm} \rightarrow$ position vector of CM.



$$\vec{F}_{21} = -\vec{F}_{12}$$

$$\checkmark \vec{F}_{21} + \vec{F}_{12} = 0$$

$$\begin{cases} \vec{p} = m\vec{v} \\ \vec{v} = \frac{d\vec{r}}{dt} \\ \vec{a} = \frac{d\vec{v}}{dt} \\ \vec{a} = \frac{d}{dt}\left(\frac{d\vec{r}}{dt}\right) \\ \vec{a} = \frac{d^2 \vec{r}}{dt^2} \end{cases}$$