ISC XI, Centre of Mass

Saturday, October 16, 2021 7:55 PM

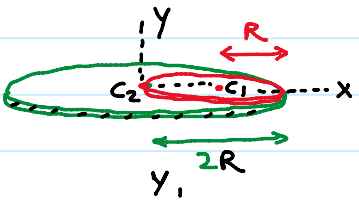
$m_1, m_2, m_3 \ldots\ldots$

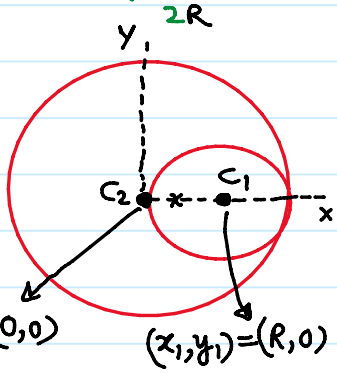
$(x_1,y_1,z_1), (x_2,y_2,z_2) \ldots$

$$x_{cm} = \frac{m_1x_1 + m_2x_2 + \ldots}{m_1+m_2+\ldots} \quad ; \quad y_{cm} = \frac{m_1y_1+m_2y_2+\ldots}{m_1+m_2+\ldots}$$

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots}{m_1+m_2+\ldots} \Rightarrow \vec{r}_{cm} = \frac{1}{M}\sum_{i=1}^{n} m_i\vec{r}_i$$

A uniform disc of radius R is put over another uniform disc of radius 2R of the same thickness and density. The peripheries of the two discs touch each other. Located the centre of mass of the system.





$m_1 = (\pi R^2 d)\rho$

$m_2 = \pi\cdot(2R)^2 d\rho = 4\pi R^2 d\rho$

$m_2 = 4m_1$

$$x_{cm} = \frac{m_1x_1+m_2x_2}{m_1+m_2} = \frac{m_1\times R + 4m_1\times 0}{m_1+4m_1}$$

$$x_{cm} = \frac{m_1R}{5m_1} \Rightarrow x_{cm} = R/5$$

$$y_{cm} = \frac{m_1y_1+m_2y_2}{m_1+m_2} \Rightarrow y_{cm} = \frac{0+0}{m_1+4m_1} = 0$$

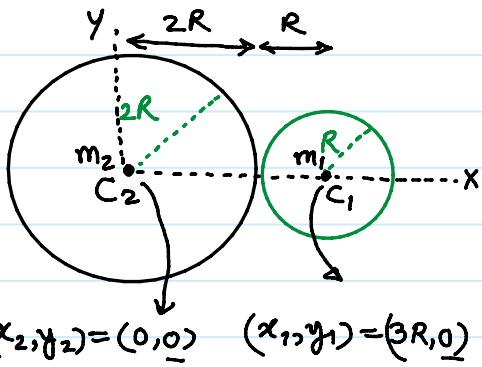
$(x_{cm}, y_{cm}) = \left(\frac{R}{5}, 0\right)$

$m_1 = A_1\cdot k = \pi R^2 k$

$m_2 = A_2 k = \pi(2R)^2 k = 4\pi R^2 k$

$m_2 = 4m_1$

Two uniform Spheres of same material but of radius R and 2R are kept in contact. Find out the distance of the Centre of Mass from the centre of the larger Sphere.



$m_1 = \frac{4}{3}\pi R^3\rho$

$m_2 = \frac{4}{3}\pi(2R)^3\rho = 8\left(\frac{4}{3}\pi R^3\rho\right)$

$m_2 = 8m_1$

$$x_{cm} = \frac{m_1x_1+m_2x_2}{m_1+m_2}$$

$$= \frac{m_1\times 3R + 8m_1\times 0}{m_1+8m_1} = \frac{3m_1R}{9m_1} \Rightarrow \boxed{x_{cm} = \frac{R}{3}}$$