NUCLEI

1. Atomic Masses and Composition of Nucleus

- **Atomic Mass Unit (u):** Since the mass of an atom is extremely small, it is measured in atomic mass units. One atomic mass unit is defined as $1/12^{\text{th}}$ of the mass of an atom of the carbon isotope ^{12}C.
  - **Value:** $1 \text{ u} = 1.660539 \times 10^{-27} \text{ kg}$.
  - The mass of an atom is measured using a **mass spectrometer**.
- **Composition of the Nucleus:** A nucleus consists of protons and neutrons, collectively known as **nucleons**.
  - **Protons:** Discovered by Rutherford. It carries a positive charge $+e$. Its mass m_p is approximately 1.00727 u.
  - **Neutrons:** Discovered by James Chadwick in 1932. It is electrically neutral. Its mass m_n is approximately 1.00866 u.
- **Atomic Number (Z):** The number of protons in the nucleus.
- **Mass Number (A):** The total number of protons (Z) and neutrons (N).
  - **Formula:** $A = Z + N$
- **Nuclear Species:**
  - **Isotopes:** Nuclei with the same Z but different A (e.g., $^1_1\text{H}, ^2_1\text{H}, ^3_1\text{H}$).
  - **Isobars:** Nuclei with the same A but different Z (e.g., ^3_1H and ^3_2He).
  - **Isotones:** Nuclei with the same number of neutrons N.

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2. Size and Density of the Nucleus

- **Nuclear Radius (R):** The volume of a nucleus is proportional to its mass number A. Therefore, the radius is proportional to $A^{1/3}$.
  - **Formula:**

$$R = R_0 A^{1/3}$$

  - Where $R_0 \approx 1.2 \times 10^{-15}$ m (or 1.2 fm).
- **Nuclear Density (ρ):** Density is the ratio of mass to volume.
  - **Formula:** $\rho = \frac{\text{Mass}}{\text{Volume}} = \frac{A \cdot m}{\frac{4}{3}\pi R_0^3 A} = \frac{3m}{4\pi R_0^3}$
  - **Key Property:** The mass number A cancels out, meaning **nuclear density is constant** for all nuclei, regardless of their size. It is roughly $2.3 \times 10^{17} \text{ kg/m}^3$.

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3. Mass-Energy and Binding Energy

- **Mass-Energy Equivalence:** Einstein's famous relation states that mass can be converted into energy and vice versa.
  - **Formula:**

$$E = mc^2$$

  - **Energy of 1 u:** $1 \text{ u} \times c^2 \approx 931.5 \text{ MeV}$.

- **Mass Defect (Δm):** The difference between the sum of the masses of the individual nucleons and the actual mass (M) of the nucleus.
  - **Formula:**

$$\Delta m = [Zm_p + (A - Z)m_n] - M$$

- **Binding Energy (E_b):** The energy equivalent of the mass defect. It is the energy required to separate a nucleus into its individual nucleons.
  - **Formula:**

$$E_b = \Delta mc^2$$ Eb (in MeV) = Δ M (in u) x 931.5

- **Binding Energy per Nucleon (E_{bn}):** This is the average energy required to remove a nucleon from the nucleus.
  - **Formula:** $E_{bn} = \frac{E_b}{A}$

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4. Characteristics of the Binding Energy Curve

The E_{bn} curve plots the binding energy per nucleon against the mass number A.

- **Peak Stability:** The curve has a maximum of about $8.75\ \text{MeV}$ for $A = 56$ (^{56}Fe). Iron is among the most stable nuclei.
- **Constancy:** For the range $30 < A < 170$, the curve is relatively flat, staying around $8\ \text{MeV}$ per nucleon.
- **Saturation:** The flatness indicates that the nuclear force is **saturated**—a nucleon only interacts with its nearest neighbors.
- **Low A and High A Drops:**
  - For $A < 30$ and $A > 170$, E_{bn} is lower, meaning these nuclei are less stable.
  - **Fusion:** Light nuclei can move toward the stable peak by joining together.
  - **Fission:** Heavy nuclei can move toward the stable peak by splitting.

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5. Nuclear Force

- **Nature:** The strongest force in nature, acting between nucleons to overcome the electrostatic repulsion of protons.
- **Range:** Very short-range, becoming negligible beyond a few femtometers (fm).
- **Distance Dependence:**
  - Attractive at distances $> 0.8\ \text{fm}$.
  - Repulsive at distances $< 0.8\ \text{fm}$ (prevents the nucleus from collapsing).

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6. Nuclear Energy Calculation Formulae

To calculate energy released (Q-value) in a nuclear reaction ($a + b \longrightarrow c + d$):

- **Using Mass:** $Q = [(\text{mass}_a + \text{mass}_b) - (\text{mass}_c + \text{mass}_d)] \times c^2$
- **Using Binding Energy:** $Q = \text{BE}_{\text{products}} - \text{BE}_{\text{reactants}}$
- If $Q > 0$, the reaction is **exothermic** (releases energy).

* Only those nuclear reactions are naturally possible for which value of Q is positive.

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7. Nuclear Fission and Chain Reaction

- **Nuclear Fission:** The process where a heavy nucleus splits into two lighter fragments when struck by a neutron.
  - **Example:** ${}^{235}_{92}\mathrm{U} + {}^{1}_{0}\mathrm{n} \rightarrow {}^{144}_{56}\mathrm{Ba} + {}^{89}_{36}\mathrm{Kr} + 3{}^{1}_{0}\mathrm{n} + Q$
  - The energy released is roughly $200\ \mathrm{MeV}$ per fission event.
- **Nuclear Chain Reaction:** The neutrons released in one fission event can trigger further fissions in nearby nuclei.
  - **Multiplication Factor (K):** $K = \frac{\text{Number of neutrons in one generation}}{\text{Number of neutrons in previous generation}}$.
  - **Critical ($K = 1$):** Reaction is steady.
  - **Supercritical ($K > 1$):** Reaction accelerates (used in weapons).
  - **Subcritical ($K < 1$):** Reaction dies out.

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8. Nuclear Reactor

A device designed to maintain a **controlled** nuclear chain reaction.

- **Moderator:** Slows down fast neutrons to "thermal" speeds to increase the probability of fission (e.g., Water, Heavy Water D_2O, Graphite).
- **Control Rods:** Made of neutron-absorbing material like Cadmium or Boron to adjust the K value.
- **Coolant:** Transfers heat from the reactor core to a heat exchanger to produce steam (e.g., Water, liquid Sodium).
- **Reflector:** Surrounds the core to reflect escaping neutrons back into the reaction area.

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9. Nuclear Fusion

- **Definition:** The process of combining light nuclei to form a heavier nucleus.
- **Mechanism:** Because nuclei are positively charged, they repel each other. To overcome this **Coulomb barrier**, they must have very high kinetic energy.
- **Thermonuclear Fusion:** Fusion occurring due to extreme thermal motion at very high temperatures ($\approx 10^7\ \mathrm{K}$).
- **Solar Energy (Proton-Proton Cycle):**
  - $4{}^{1}_{1}\mathrm{H} \rightarrow {}^{4}_{2}\mathrm{He} + 2e^{+} + 2\nu + 26.7\ \mathrm{MeV}$.
- **Fusion vs. Fission:** Fusion releases more energy per unit mass than fission and produces less radioactive waste, but it is much harder to sustain in a controlled environment on Earth.