

If the total external force acting on a system of bodies is zero (or for an isolated system) the total momentum of the system remains conserved.

Proof: Let there are two bodies A and B present an isolated system. Let they apply force on one another which are \vec{F}_{BA} & \vec{F}_{AB} . According to Newton's III law

$$\vec{F}_{BA} = -\vec{F}_{AB}$$

Or
$$\vec{F}_{BA} + \vec{F}_{AB} = 0$$

Applying Newton's II law on individual bodies.

For body A:
$$\frac{\Delta \vec{P}_A}{\Delta t} = \vec{F}_{AB} \quad \dots\dots\dots (ii)$$

For body B:
$$\frac{\Delta \vec{P}_B}{\Delta t} = \vec{F}_{BA} \quad \dots\dots\dots (iii)$$

By adding eq. (ii) and (iii)

$$\frac{\Delta \vec{P}_A}{\Delta t} + \frac{\Delta \vec{P}_B}{\Delta t} = \vec{F}_{AB} + \vec{F}_{BA}$$

Or
$$\frac{\Delta \vec{P}}{\Delta t} = 0$$

Or
$$\Delta \vec{P} = 0$$

Or
$$\vec{P} = \text{constant}$$

i.e If $\sum \vec{F}_{ext} = 0$, then total momentum of the system remains conserved.